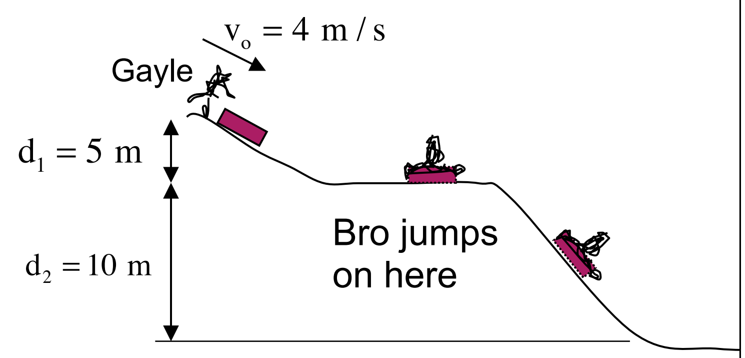
Problem 6.31

50 kg Gayle moving 4 m/s jumps on a stationary 5 kg sled and slides down a hill for a net 5 vertical meters. At that point, Gayle's 30 kg brother, also stationary, hops on and the two slide an additional 10 vertical meters. What's their speed at the bottom?



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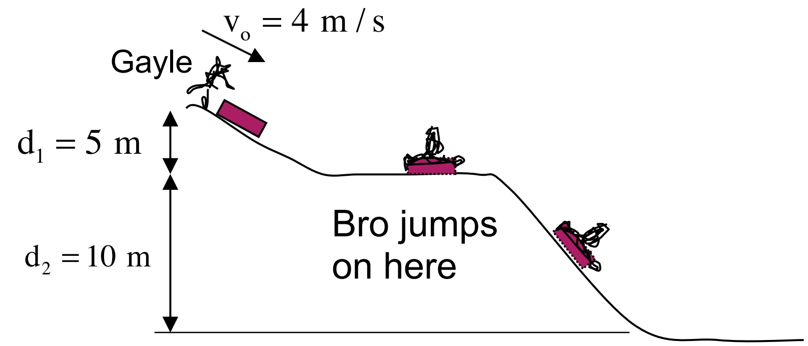


Gayle has momentum--she's moving before hopping onto the stationary sled. The sled is massive and initially has no momentum (it's stationary). The "total momentum" of the system before the collision is, therefore, all wrapped up in her initial momentum. When she jumps on the sled, it speeds up (it's momentum increases) and she slows down (her momentum decreases). Because there are no external forces acting through the collision, though, the sum of her new momentum and the sled's new momentum should equal her before-collision momentum (i.e., the total momentum in the system). Conservation of momentum can therefore be written as:

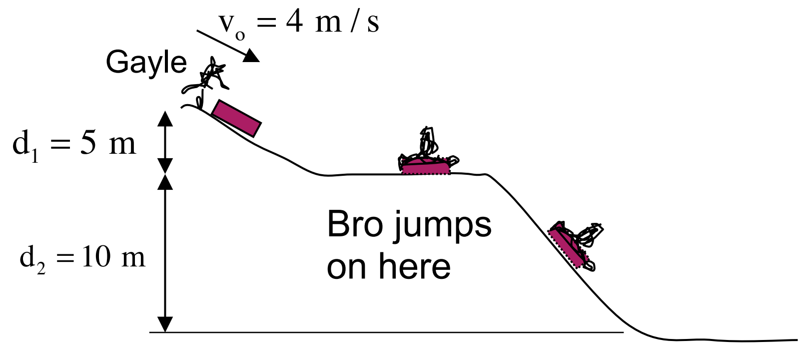
$$\sum p_{o,x} + \sum F_{ext,x}\Delta t = \sum p_{f,x}$$

$$m_{gayle}(v_o) + 0 = m_{gayle}v_1 + m_{sled}v_1$$

$$(50\ kg)(4\ m/s) + 0 = (50\ kg)v_1 + (5\ kg)v_1$$

$$\Rightarrow \quad v_1 = 3.64\ m/s$$

Energy is not conserved when Gayle throws herself on the sled, but after she and the sled hit their 3.64 m/s, the two will not be under the influence of any extraneous forces in the direction of motion and conservation of energy can be used until "bro" jumps on. That is:



$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$

$$\frac{1}{2}(m_{gayle} + m_{sled})(v_1)^2 + (m_{gayle} + m_{sled})(gy_o) + 0 = \frac{1}{2}(m_{gayle} + m_{sled})(v_2)^2 + (m_{gayle} + m_{sled})(gy_2)$$

$$\frac{1}{2}(50\ kg + 5\ kg)(3.64\ m/s)^2 + (50\ kg + 5\ kg)(9.8\ m/s^2)(15\ m) + 0 = \frac{1}{2}(50\ kg + 5\ kg)(v_2)^2 + (50\ kg + 5\ kg)(9.8\ m/s^2)(10\ m)$$

$$\Rightarrow \quad v_2 = 10.5\ m/s$$

Energy is not conserved when little brother decides to join in with Gayle, but as there are not external forces acting through the collision and along the line of motion as he jumps, momentum is conserved. Following through with that yields:

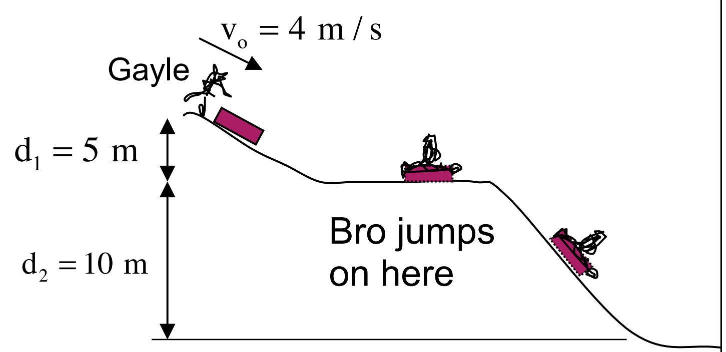
$$\sum p_{o,x} + \sum F_{ext,x}\Delta t = \sum p_{f,x}$$

$$(m_{gayle} + m_{sled})(v_2) + 0 = (m_{gayle} + m_{sled} + m_{bro})v_3$$

$$(55\ kg)(10.5\ m/s) + 0 = (85\ kg)v_3$$

$$\Rightarrow \quad v_3 = 6.82\ m/s$$

Once everyone is on board and any energy loss has been incurred due to “bro’ s” hopping on board, energy will be conserved from there on. Using that yields:



$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$

$$\frac{1}{2}m_{everything}(v_3)^2 + \left(m_{everything}gd_1\right) + 0 = \frac{1}{2}m_{everything}(v_{bot})^2 + 0$$

$$\frac{1}{2}(85\text{ kg})(6.82\text{ m/s})^2 + (85\text{ kg})\left(9.8\text{ m/s}^2\right)(10\text{ m}) + 0 = \frac{1}{2}(85\text{ kg})(v_{bot})^2 + 0$$

$$\Rightarrow \quad v_{bot} = 15.6\text{ m/s}$$